Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Sample Assessment Materials for first teaching September 2018

(Time: 1 hour 30 minutes)	Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Answer ALL questions. Write your answers in the spaces provided.

1. $$f(x) = x^4 + x^3 + 2x^2 + ax + b,$$

where a and b are constants.

When f(x) is divided by (x – 1), the remainder is 7

(a) Show that $a + b = 3$ **(2)**

$= (x-(-2))$

When f(x) is divided by (x + 2), the remainder is –8

(b) Find the value of a and the value of b **(5)**

(a) USE : $\boxed{\text{REMAINDER THEOREM: if } \frac{f(x)}{(x-k)} = \text{quotient} + \text{remainder} \text{: then } f(k) = \text{remainder}}$

if $\frac{f(x)}{x-1} = \text{quotient} + 7$

then $f(1) = 7$

• Substitute $x = 1$ in $f(x) = 7$

$f(1) = 1^4 + 1^3 + 2(1)^2 + a(1) + b = 7$

$1 + 1 + 2 + a + b = 7$

$4 + a + b = 7$

$a + b = 7 - 4$

$a + b = 3$

(b) USE REMAINDER THEOREM AGAIN :

if $\frac{f(x)}{x-(-2)} = \text{quotient} - 8$

$=$ then $f(-2) = -8$

Leave blank

Question 1 continued

• Substitute $x=-2$ in $f(x)=-8$

$f(-2) = (-2)^4 + (-2)^3 + 2(-2)^2 + a(-2) + b = -8$

$16 - 8 + 8 - 2a + b = -8$

$16 - 2a + b = -8$

$-2a + b = -8-16$

$-2a + b = -24$

Now we have 2 equations :

① $a + b = 3$

② $-2a + b = -24$

SOLVE SIMULTANEOUSLY :

(a) ELIMINATION OR (b) SUBSTITUTION :

(a)

$[a + b = 3] \times 2$

$2a + 2b = 6$ ⊕

$-2a + b = -24$

$3b = -18$

$b = -6$

$a + (-6) = 3$

$a = 9$

(b)

$a + b = 3$

$a = 3 - b$

$-2a + b = -24$

$-2(3-b) + b = -24$

$-6 + 2b + b = -24$

$3b = -18$

$b = -6$

$a = 3 - (-6)$

$a = 9$

So : $\boxed{\begin{matrix} a = 9 \\ b = -6 \end{matrix}}$

Q1

(Total for Question 1 is 7 marks)

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2. The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$. The sum to infinity of the series is S_∞

a_1 r

(a) Find the value of S_∞

(2)

The sum to N terms of the series is S_N partial sum

(b) Find, to 1 decimal place, the value of S_{12}

(2)

(c) Find the smallest value of N, for which $S_\infty - S_N < 0.5$

(4)

(a) For a geometric series,

$$\boxed{S_\infty = \frac{a_1}{1-r}}$$

$$S_\infty = \frac{20}{1-7/8} = 160$$

(b) Use partial sum formula:

$$S_N = \frac{a_1(1-r^N)}{(1-r)} \quad \text{where } N=12$$

$$S_{12} = \frac{20(1-(7/8)^{12})}{1-7/8} = 127.77324\ldots$$
$$= 127.8 \text{ (1 d.p.)}$$

(c) $S_\infty = 160$ (found in (a))

$$S_N = \frac{20(1-(7/8)^N)}{(1-7/8)}$$

$$160 - \frac{20(1-(7/8)^N)}{(1-7/8)} < 0.5$$

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Question 2 continued

$$160 - \frac{20\left(1-\left(\frac{7}{8}\right)^N\right)}{1/8} < 0.5$$

$$160 - 160\left(1-\left(\frac{7}{8}\right)^N\right) < 0.5$$

$$160\left(1 - 1 - \left(\frac{7}{8}\right)^N\right) < 0.5$$

$$\left(160\left(\frac{7}{8}\right)^N < 0.5\right) \div 160$$

$$\log\left(\frac{7}{8}\right)^N < \log\frac{0.5}{160}$$

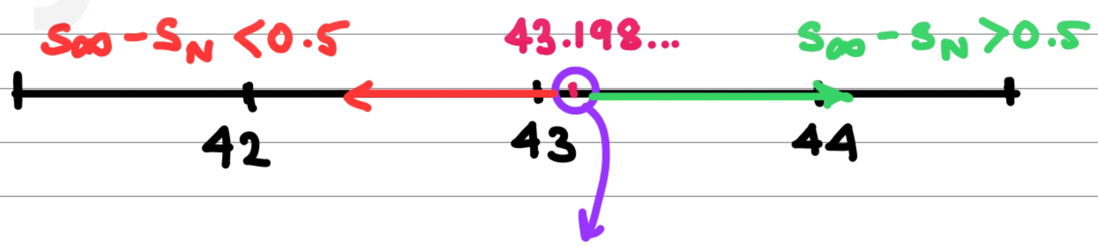
$\log(7/8)$ is NEGATIVE!

$$N \log\frac{7}{8} < \log\left(\frac{0.5}{160}\right)$$

$$N > \frac{\log(0.5/160)}{\log(7/8)}$$

INVERT THE SIGN! multiplying by $-$ve no. = INVERT THE SIGN !!

$$N > 43.198\ldots$$



$S_\infty - S_N = 0.5$

So round UP : $N = 44$

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Question 2 continued

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Question 2 continued

Q2

(Total for Question 2 is 8 marks)

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3. $$y = \sqrt{(3^x + x)}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

Interval btwn two x-coords = h of trapezium

x	0	0.25	0.5	0.75	1
y	y_1 1	1.251	1.494	1.741	2 y_5

multiplied by 2 as all are shared by two trapezia

(2)

(b) Use the trapezium rule with all the values of y from your table to find an approximation for the value of

$$\int_0^1 \sqrt{(3^x + x)}\, dx$$

You must show clearly how you obtained your answer.

(4)

(c) Explain how the trapezium rule could be used to obtain a more accurate estimate for the value of

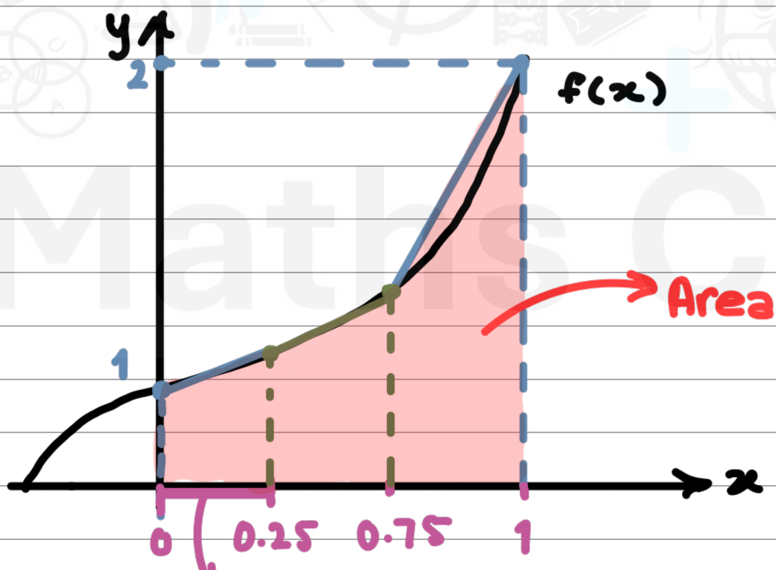
$$\int_0^1 \sqrt{(3^x + x)}\, dx$$

(1)

(a) $y = \sqrt{3^{0.5} + 0.5} = 1.494$ (3 dp)

$y = \sqrt{3^{0.75} + 0.75} = 1.741$ (3 dp)

(b)



$h = 0.25 - 0 = 0.25$

(Formula is given in formula booklet. The graph is only a visual representation. DO NOT DRAW A GRAPH IN THE EXAM)

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Question 3 continued

$$A = \frac{1}{2}h\left[y_1 + 2(y_2 + y_3 + y_4) + y_5\right]$$

$$A = \frac{1}{2}(0.25)\left[1 + 2(1.251 + 1.494 + 1.741) + 2\right]$$

$$A = 0.125(11.966) = 1.49575$$

$$A = 1.50 \text{ (3 s.f.)}$$

(c) As we can see in the graph, using trapezia can mean over or underestimating area. To make the trapezia fit closer with the curve f(x), we can:

- decrease width of strips
- use more trapezia

(same idea.)

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Question 3 continued

Q3

(Total for Question 3 is 7 marks)

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natural numbers : 1,2,3,4,5...

4. Given $n \in \mathbb{N}$, prove, by exhaustion, that $n^2 + 2$ is not divisible by 4.

(4)

We can go through a series of natural numbers and "exhaust" them

n	n^2	n^2+2	even/odd?	divisible by 4 ?
1	1	3	odd	NO :(
2	4	6	even	NO :(
3	9	11	odd	NO :(
4	16	18	even	NO :(
5	25	27	odd	NO :(
6	36	38	even	NO :(

Why ?

When n is odd, n^2 and n^2+2 are both odd. Odd numbers cannot be divisible by 4.

When n is even, n^2 is even and ALWAYS a multiple of 4. So, adding 2 creates a remainder ; n^2+2 cannot be a multiple of 4.

So, for all n, n^2+2 cannot be divisible by 4.

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Question 4 continued

OR : ALGEBRAIC PROOF :

If n is even : $n = 2k$

$$(2k)^2 + 2 = 4k^2 + 2$$

$$\frac{4k^2+2}{4}$$

$$= k^2 + \frac{1}{2}$$

remainder; not divisible by 4 :(

If n is odd : $n = 2k+1$

$$(2k+1)^2 + 2 = 4k^2 + 4k + 1 + 2$$

$$= \frac{4k^2 + 4k + 3}{4}$$

$$= k^2 + k + \frac{3}{4}$$

remainder; not divisible by 4 :(

Q4

(Total for Question 4 is 4 marks)

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5. An arithmetic series has first term a and common difference d.

(a) Prove that the sum of the first n terms of the series is

$$\frac{1}{2}n[2a + (n - 1)d]$$

(4)

A company, which is making 200 mobile phones each week, plans to increase its production.

d a_1

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week N.

a_n

(b) Find the value of N

(2)

The company then plans to continue to make 600 mobile phones each week.

(c) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1.

(5)

(a) In an arithmetic sequence, we add d each term to find the next term.

So:

$S_n = a + a+d + \dots + a+(n-2)d + a+(n-1)d$

If we write the sum backwards:

$S_n = a+(n-1)d + a+(n-2)d + \dots + a+d + a$

Now, we can add both to find double S_n:

$+$ $S_n = a + a+d + \dots + a+(n-2)d + a+(n-1)d$

$S_n = a+(n-1)d + a+(n-2)d + \dots + a+d + a$

$2S_n = 2a+(n-1)d + 2a+(n-1)d \quad 2a+(n-1)d + \underbrace{2a+(n-1)d}_{\text{term } a_n}$

$2S_n = n(2a+(n-1)d)$

$\therefore S_n = \frac{n}{2}(2a+(n-1)d)$

Leave blank

Question 5 continued

(b) $a_n = a_1 + d(n-1)$

$600 = 200 + 20(n-1)$

$\frac{400}{20} = \frac{20(n-1)}{20}$

$20 = n-1$

$n = 21$

(c)

ARITHMETIC SERIES — CONSTANT

WEEK NO. :	1, 2, 3, ..., 21,	22, 23 52
PHONES :	200, 220, 240, ... , 600,	600, 600, ..., 600

use PARTIAL SUM FORMULA

normal multiplication!

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{21}{2}[2(200) + (21-1)(20)]$

$= 8400$ phones in 21 weeks!

no. weeks left $= 52 - 21 = 31$

$600 \times 31 = 18\,600$

18 600 phones in 31 weeks!

So total no. phones $= 8400 + 18600$
$= 27\,000$

Leave blank

Question 5 continued

Q5

(Total for Question 5 is 11 marks)

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6. (i) Find the exact value of x for which

$$\log_2(2x) = \log_2(5x + 4) - 3$$

(4)

(ii) Given that

$$\log_a y + 3\log_a 2 = 5$$

express y in terms of a. Give your answer in its simplest form.

(3)

Recall rules of logarithms :

$\log_a(b) + \log_a(c) = \log_a(b \times c)$

$\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right)$

$\log_a b = c \rightarrow b = a^c$

$\log a^b = b\log a$

(i) $\log_2(2x) = \log_2(5x+4) - 3$

$\log_2(5x+4) - \log_2(2x) = 3$

$\log_2\left(\frac{5x+4}{2x}\right) = 3$

$2x \times \left(\frac{5x+4}{2x}\right) = (2^3) \times 2x$

$5x + 4 = 16x$

$4 = 11x$

$x = \frac{4}{11}$

Leave blank

Question 6 continued

ALTERNATIVE :

$$\log_2(2x) = \log_2(5x+4) - 3$$

$$\log_2(2x) - \log_2(5x+4) = -3$$

$$\log_2\left(\frac{2x}{5x+4}\right) = -3$$

$$5x+4 \times \left(\frac{2x}{5x+4}\right) = (2^{-3}) \times (5x+4)$$

$$8 \times (2x) = \left(\frac{1}{8}(5x+4)\right) \times 8$$

$$16x = 5x + 4$$

$$11x = 4$$

$$x = \frac{4}{11}$$

(ii) $$\log_a y + 3\log_a 2 = 5$$

$$\log_a y + \log_a 2^3 = 5$$

$$\log_a (y \times 8) = 5$$

$$\frac{8y}{8} = \frac{a^5}{8}$$

$$y = \frac{1}{8}a^5$$

Q6

(Total for Question 6 is 7 marks)

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7.

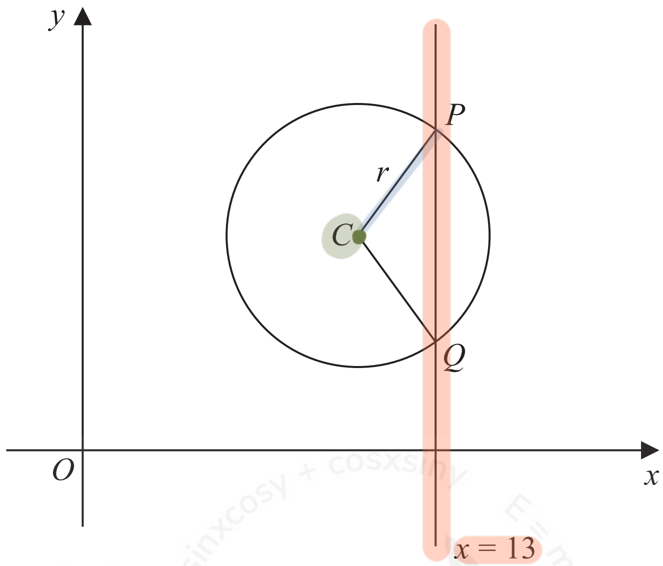

Figure 1

The circle with equation

$$x^2 + y^2 - 20x - 16y + 139 = 0$$

had centre C and radius r.

(a) Find the coordinates of C. **(2)**

(b) Show that $r = 5$ **(2)**

The line with equation $x = 13$ crosses the circle at the points P and Q as shown in Figure 1.

(c) Find the y coordinate of P and the y coordinate of Q. **(3)**

A tangent to the circle from O touches the circle at point X.

(d) Find, in surd form, the length OX. **(3)**

(a) Standard form of equation of a circle:

$$(x - x_{\text{centre}})^2 + (y - y_{\text{centre}})^2 = r^2$$

↳ rewrite given eq. into standard form:

$$x^2 - 20x + y^2 - 16y + 139 = 0$$

Leave blank

Question 7 continued

(1) complete the square for terms with x :

$$x^2 - 20x + y^2 - 16y = -139$$

↳ add $\left(\frac{b}{2}\right)^2$ to both sides :

$$x - 20x + \left(\frac{-20}{2}\right)^2 + y^2 - 16y = -139 + \left(\frac{-20}{2}\right)^2$$

$$x - 20x + 100 + y^2 - 16y = -139 + 100$$

$$x - 20x + 100 + y^2 - 16y = -39$$

↳ factorise

$$(x-10)^2 + y^2 - 16y = -39$$

(3) complete the square for terms with y :

$$(x-10)^2 + y^2 - 16y = -39$$

↳ add $\left(\frac{b}{2}\right)^2$ to both sides :

$$(x-10)^2 + y^2 - 16y + \left(\frac{-16}{2}\right)^2 = -39 + \left(\frac{-16}{2}\right)^2$$

$$(x-10)^2 + y^2 - 16y + 64 = -39 + 64$$

$$(x-10)^2 + y^2 - 16y + 64 = 25$$

↳ factorise

$$(x-10)^2 + (y-8)^2 = 25$$

So : $(x-10)^2 + (y-8)^2 = 25$

coords of centre $= (10, 8)$

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Question 7 continued

(b) in (a), we found

$$(x-10)^2+(y-8)^2=25$$

$$(x-x_{centre})^2+(y-y_{centre})^2=r^2$$

$r^2=25$ so $r=\sqrt{25}=5$

(c) $(x-10)^2+(y-8)^2=25$

so, at $x=13$:

$$(13-10)^2+(y-8)^2=25$$

$$(3)^2+(y^2-16y+64)=25$$

$$73+y^2-16y=25$$

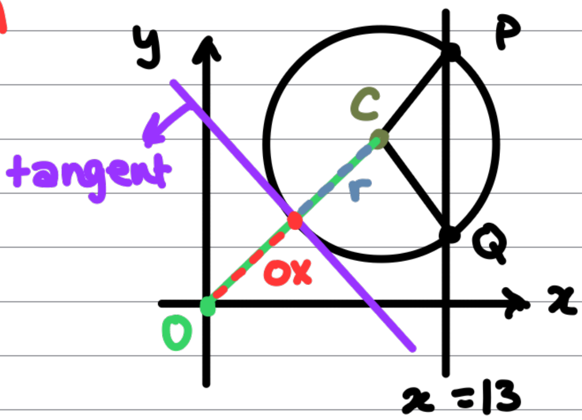
$$y^2-16y+48=0 \rightarrow \text{factorise!}$$

$$(y-4)(y-12)=0$$

$y-4=0$ or $y-12=0$

$y=4$ or $y=12$

(d) IMAGINE A DIAGRAM:



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Question 7 continued

where $OX = OC - r$

$$\boxed{\begin{array}{l}\text{RECALL : distance formula :} \\ d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\end{array}}$$

$OC = \sqrt{(10-0)^2 + (8-0)^2} = \sqrt{164}$

$OX = \sqrt{164} - 5 = \sqrt{139}$

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Q7

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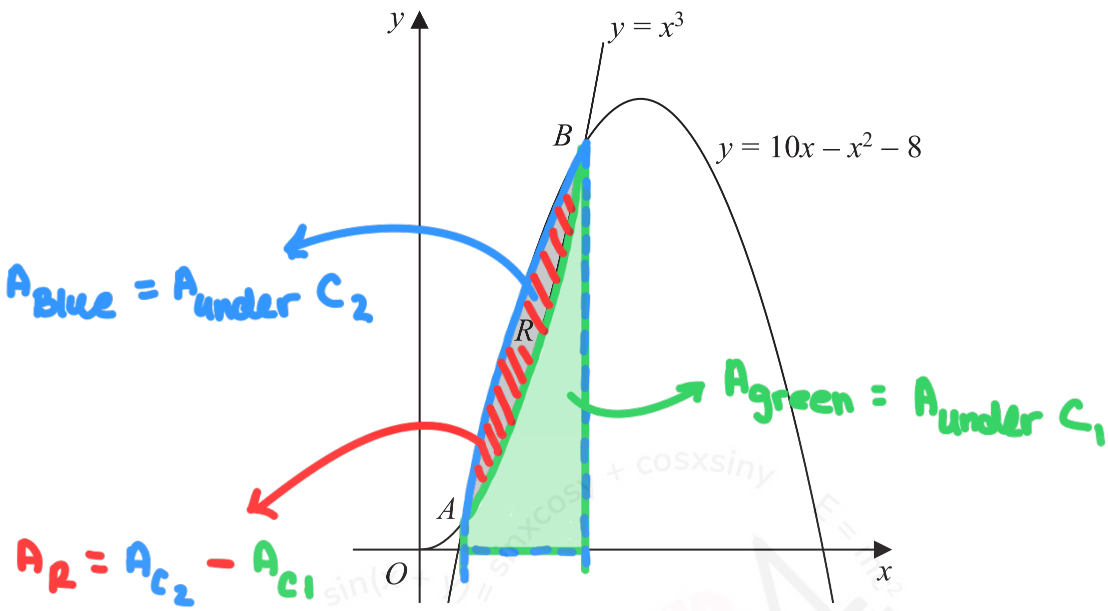

Figure 2

Figure 2 shows a sketch of part of the curves C_1 and C_2 with equations

$$C_1: y = 10x - x^2 - 8 \qquad x > 0$$

$$C_2: y = x^3 \qquad x > 0$$

The curves C_1 and C_2 intersect at the points A and B.

(a) Verify that the point A has coordinates (1, 1) **(1)**

(b) Use algebra to find the coordinates of the point B **(6)**

The finite region R is bounded by C_1 and C_2

(c) Use calculus to find the exact area of R **(5)**

(a) A is shared by C_1 and C_2

$\therefore\ y_{C_1} = y_{C_2}$ at x_A

when $x = 1$:

C_1 : $y = 10(1) - (1)^2 - 8 = 10 - 1 - 8 = 1$

C_2 : $y = (1)^3 = 1$

So : coords $A = (1, 1)$

Leave blank

Question 8 continued

(b) B is shared by C_1 and C_2

$\therefore \; y_{C_1} = y_{C_2}$ at x_B

$10x - x^2 - 8 = x^3$

$x^3 + x^2 - 10x + 8 = 0$

USING FACTOR THEOREM, we know :
If $x = 1$ is a solution of $y_{C_1} = y_{C_2}$, then $(x-1)$ is a factor of $x^3 + x^2 - 10x + 8 = 0$

Use algebraic division to factorise :

$$x - 1 \,\big)\, \overline{x^3 + x^2 - 10x + 8}$$

(1) Divide first term of dividend by first term of divisor : $\frac{x^3}{x} = x^2 \rightarrow$ first term of quotient

$$\begin{array}{r l} & x^2 \\ x - 1 \,\big) & \overline{x^3 + x^2 - 10x + 8} \end{array}$$

$\div$

(2) Multiply out and write under :

$$\begin{array}{r l} & x^2 \\ x - 1 \,\big) & \overline{x^3 + x^2 - 10x + 8} \\ & x^3 - x^2 \\ \hline \end{array}$$

(3) Subtract then bring down next term :

$$\begin{array}{r l} & x^2 \\ x - 1 \,\big) & \overline{x^3 + x^2 - 10x + 8} \\ & -(x^3 - x^2) \downarrow \\ \hline & 2x^2 - 10x \end{array}$$

Question 8 continued

(4) Divide first term of expression by first term of divisor : $\frac{2x^2}{x} = 2x \rightarrow$ next term of quotient

$$
\begin{array}{r|l}
 & x^2 + 2x \\
\hline
x-1 & x^3 + x^2 - 10x + 8 \\
 & -(x^3 - x^2) \\
\hline
 & 2x^2 - 10x
\end{array}
$$

(5) Multiply out and write under :

$$
\begin{array}{r|l}
 & x^2 + 2x \\
\hline
x-1 & x^3 + x^2 - 10x + 8 \\
 & -(x^3 - x^2) \\
\hline
 & 2x^2 - 10x \\
 & -2x^2 - 2x \\
\hline
\end{array}
$$

(6) Subtract then bring down next term :

$$
\begin{array}{r|l}
 & x^2 + 2x \\
\hline
x-1 & x^3 + x^2 - 10x + 8 \\
 & -(x^3 - x^2) \\
\hline
 & 2x^2 - 10x \\
 & -(-2x^2 - 2x) \\
\hline
 & -8x + 8
\end{array}
$$

(7) Divide first term of expression by first term of divisor : $\frac{-8x}{x} = -8 \rightarrow$ next term of quotient

$$
\begin{array}{r|l}
 & x^2 + 2x - 8 \\
\hline
x-1 & x^3 + x^2 - 10x + 8 \\
 & -(x^3 - x^2) \\
\hline
 & 2x^2 - 10x \\
 & -(-2x^2 - 2x) \\
\hline
 & -8x + 8
\end{array}
$$

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Leave blank

Question 8 continued

(8) Multiply out and write under:

$$
\begin{array}{r|l}
 & x^2 + 2x - 8 \\
\hline
x - 1 & x^3 + x^2 - 10x + 8 \\
 & -(x^3 - x^2) \\
\hline
 & 2x^2 - 10x \\
 & -(-2x^2 - 2x) \\
\hline
 & -8x + 8 \\
 & -8x + 8
\end{array}
$$

(9) Subtract

$$
\begin{array}{r|l}
 & x^2 + 2x - 8 \\
\hline
x - 1 & x^3 + x^2 - 10x + 8 \\
 & -(x^3 - x^2) \\
\hline
 & 2x^2 - 10x \\
 & -(-2x^2 - 2x) \\
\hline
 & -8x + 8 \\
 & -(-8x + 8) \\
\hline
 & 0
\end{array}
$$

so; $y_{C_1} = y_{C_2}$ can be written as:

$f(x) = (x-1)(x^2 + 2x - 8) \rightarrow$ factorise further!

$= (x-1)(x+4)(x-2)$

so: $x = 1 \longrightarrow$ at A

$x = -4 \longrightarrow$ out of range; $x > 0$

$x = 2$

so $x_B = 2$ and $y_B = 2^3 = 8$

$\longrightarrow B = (2, 8)$

Q8

(Total for Question 8 is 12 marks)

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Question 8 continued

(c) A_R has boundaries A, B

$$A_R = A_{C_2} - A_{C_1}$$

$$A_R = \int_1^2 10x - x^2 - 8 \, dx - \int_1^2 x^3 \, dx$$

COMBINE USING RULE:

$$\boxed{\int f(x)\,dx \pm \int g(x)\,dx = \int f(x) \pm g(x)\,dx}$$

$$A_R = \int_1^2 10x - x^2 - 8 - x^3 \, dx$$

$$A_R = \int_1^2 10x - x^2 - 8 - x^3 \, dx$$

REMEMBER!

$$\boxed{\int x^n \, dx = \frac{x^{n+1}}{n+1}}$$

$$A_R = \left[\frac{10x^{1+1}}{1+1} - \frac{x^{2+1}}{2+1} - \frac{8x^{0+1}}{0+1} - \frac{x^{3+1}}{3+1} \right]_1^2$$

$$= \left[5x^2 - \frac{x^3}{3} - 8x - \frac{x^4}{4} \right]_1^2$$

$$= \left(5(2)^2 - \frac{2^3}{3} - 8(2) - \frac{(2)^4}{4}\right) - \left(5(1)^2 - \frac{1^3}{3} - 8(1) - \frac{1^4}{4}\right)$$

$$A_R = \left(20 - \frac{8}{3} - 16 - 4\right) - \left(5 - \frac{1}{3} - 8 - \frac{1}{4}\right) = \frac{11}{12}$$

$$A_R = 11/12$$

note: in ms two ways are shown. Way 2 involves an identity in the P3 syllabus, not P2, hence I have not included it.

Leave blank

9. (i) Solve, for $0 \leqslant \theta < \pi$, the equation

$$\sin 3\theta - \sqrt{3}\cos 3\theta = 0$$

giving your answers in terms of π → in radians!!

(3)

(ii) Given that

$$4\sin^2 x + \cos x = 4 - k, \quad 0 \leqslant k \leqslant 3$$

(a) find $\cos x$ in terms of k

(3)

in degrees!!

(b) When $k = 3$, find the values of x in the range $0 \leqslant x < 360°$

(3)

(i) $\sin 3\theta - \sqrt{3}\cos 3\theta = 0$

$$\frac{\sin 3\theta - \sqrt{3}\cos 3\theta}{\cos 3\theta} = \frac{0}{\cos 3\theta}$$

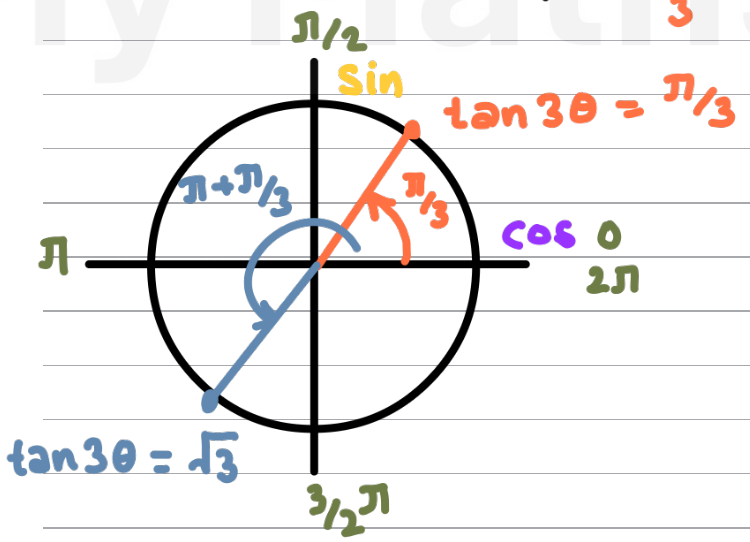
$$\frac{\sin 3\theta}{\cos 3\theta} - \frac{\sqrt{3}\,\cancel{\cos 3\theta}}{\cancel{\cos 3\theta}} = 0 \rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\tan 3\theta - \sqrt{3} = 0$

$\tan 3\theta = \sqrt{3}$, range : $3(0) < 3\theta < 3\pi$

ON CALCULATOR:

$3\theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$



So:

$3\theta = \pi/3$

$3\theta = \pi + \pi/3 = 4\pi/3$

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Question 9 continued

ALSO, since our range goes up to 3π;

$3\theta = \pi/3$

$3\theta = \pi/3 + 2\pi = 7\pi/3$

So : $\frac{3\theta}{3} = \frac{\pi}{3\times3}, \frac{4\pi}{3\times3}, \frac{7\pi}{3\times3}$

$\theta = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}$

(ii) $4\sin^2 x + \cos x = 4 - k$

$$\boxed{\text{Use identity : } \sin^2 x + \cos^2 x = 1 \quad \sin^2 x = 1 - \cos^2 x}$$

$4(1 - \cos^2 x) + \cos x = 4 - k$

$\cancel{4} - 4\cos^2 x + \cos x = \cancel{4} - k$

$4\cos^2 x - 1\cos x - k = 0$ → FACTORISE

$\cos x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\cos x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-k)}}{2(4)}$

$\cos x = \frac{1 + \sqrt{1 + 16k}}{8}$ or $\frac{1 - \sqrt{1 + 16k}}{8}$

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DO NOT WRITE IN THIS AREA

Question 9 continued

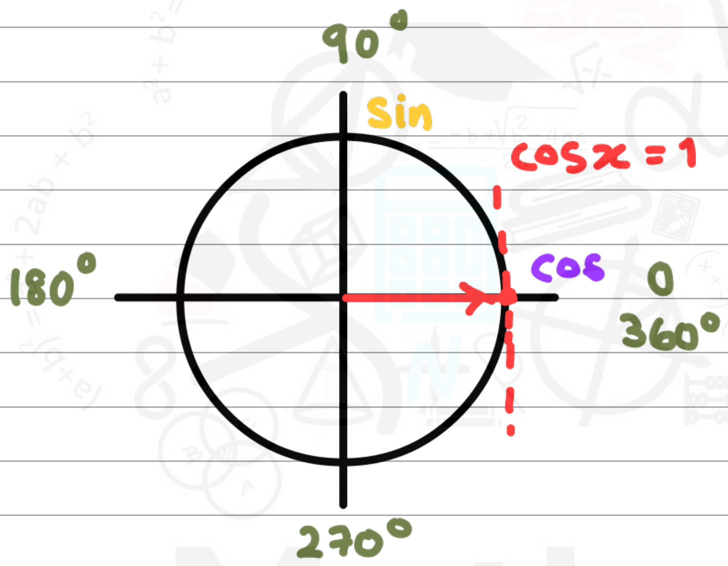
(b) when $k = 3$:

$$\cos x = \frac{1 + \sqrt{1 + 16(3)}}{8} = \frac{1 + \sqrt{49}}{8} = \frac{1+7}{8} = 1$$

$$\cos x = \frac{1 - \sqrt{1 + 16(3)}}{8} = \frac{1 - \sqrt{49}}{8} = \frac{1-7}{8} = -\frac{3}{4}$$

① when $\cos x = 1$:

ON UNIT CIRCLE:



$\cos x = 1$ at: $x = 0$
$x = 360 \rightarrow$ OUT OF RANGE ☹

② when $\cos x = -3/4$

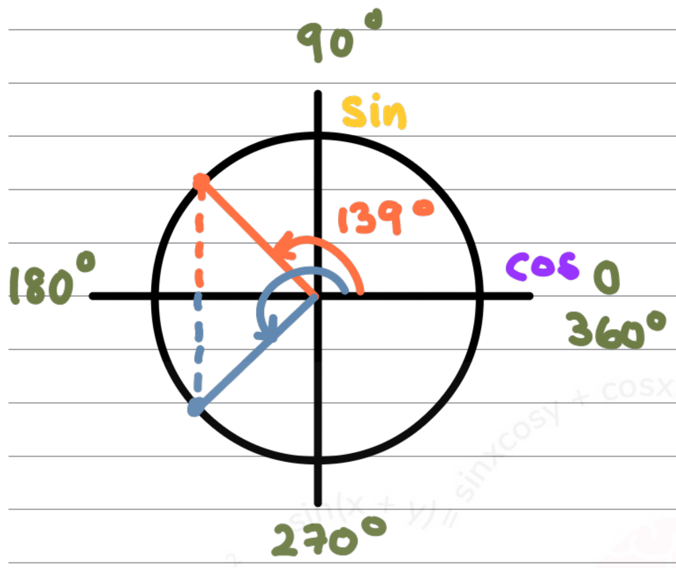
ON CALCULATOR:

$x = \cos^{-1}\left(-\frac{3}{4}\right) = 138.6° = 139°$ (3 sf).

Leave blank

Question 9 continued

ON UNIT CIRCLE :



$x = 139°$

$x = 360° - 139° = 221°$

SO : $x = 0°, 139°, 221°$

Q9

(Total for Question 9 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS